Fit for Wireless
Prepare yourself for RF-challenges
Some basics (excerpt)

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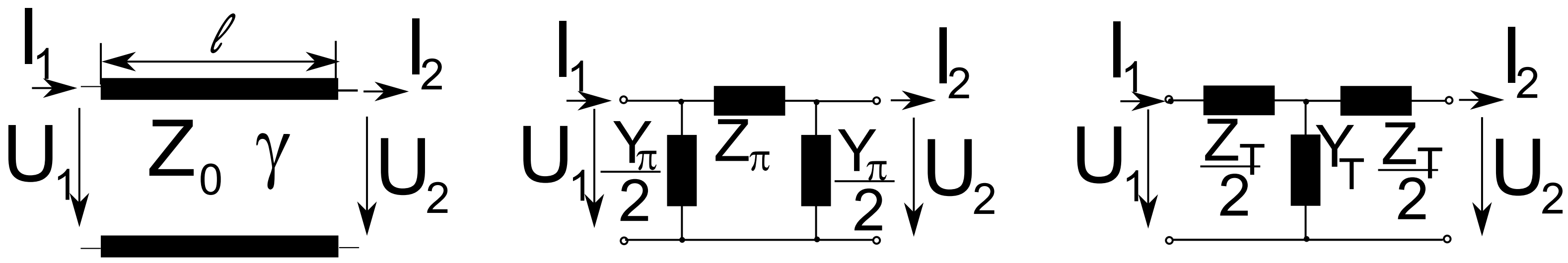
Introduction to Transmission Line Theory

Two-Port Parameters of Lines I



Transmission line and T- and Π-equivalent circuit

- A transmission line is (abstractly) nothing else than a symmetrical two-port
- All symmetrical two-ports can be expressed as T- or Π-equivalent circuits. Possibly (or rather certainly) the elements are strongly frequency dependent!

Two-Port Parameters of Lines II

- The Π-ckt has equations

$$U_2 = U_1\left(1+\frac{1}{2}Z_\pi Y_\pi\right) + Z_\pi I_1$$

$$I_2 = I_1\left(1+\frac{1}{2}Z_\pi Y_\pi\right) + U_1 Y_\pi\left(1+\frac{1}{4}Z_\pi Y_\pi\right)$$

- At the same time the line-equations in mathematical form are

$$U_2 = U_1\cosh\gamma l + Z_0 I_1 \sinh\gamma l$$

$$I_2 = I_1\cosh\gamma l + \frac{U_1}{Z_0}\sinh\gamma l$$

- Finally we find (summarized for both eq.ckt.)

$$Z_\pi = Z_0\sinh\gamma l \qquad Y_\pi = \frac{2}{Z_0}\tanh\frac{\gamma l}{2}$$

$$Y_T = \frac{1}{Z_0}\sinh\gamma l \qquad T_T = 2Z_0\tanh\frac{\gamma l}{2}$$

Electrically Short Transmission Lines

The above derived equivalent circuits are especially interesting for short transmission lines (i.e. $\gamma l \ll \lambda$)

In that case the hyperbolic functions are approximated as
$\sinh \gamma l \approx \gamma l, \quad \cosh \gamma l \approx 1, \quad \tanh \frac{\gamma l}{2} \approx \frac{\gamma l}{2}$

And so we obtain $Z_\pi \approx Z_0 \gamma l \approx j\omega\sqrt{\epsilon_r \epsilon_0 \mu_0} Z_0 l$ which looks very much like an inductance, and $Y_\pi \approx \gamma l / Z_0 \approx j\omega\sqrt{\epsilon_r \epsilon_0 \mu_0}/Z_0 l$ which looks like a capacitance.

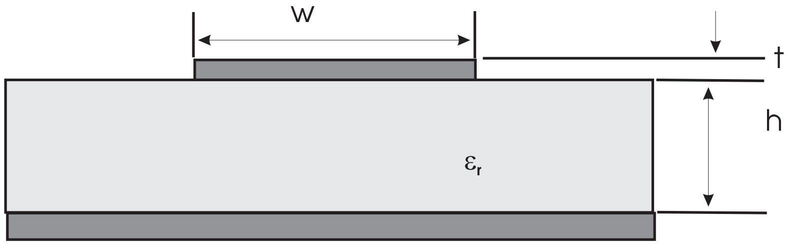
Which is dominant is mostly dependent on Z_0. For large Z_0 there is a dominant series inductance with small (parasitic) shunt capacitance. For a small Z_0 we see capacitive behavior with parasitic inductance.

Similar here are the parameters of the T-equivalent circuit: $Y_T \approx 1/Z_0 \sinh \gamma l \approx j\omega\sqrt{\epsilon_r \epsilon_0 \mu_0} Z_0$, again, this is a capacitance and also $Z_T \approx Z_0 \gamma l \approx j\omega\sqrt{\epsilon_r \epsilon_0 \mu_0}/Z_0$. The latter is now the missing (evtl. parasitic) inductance.

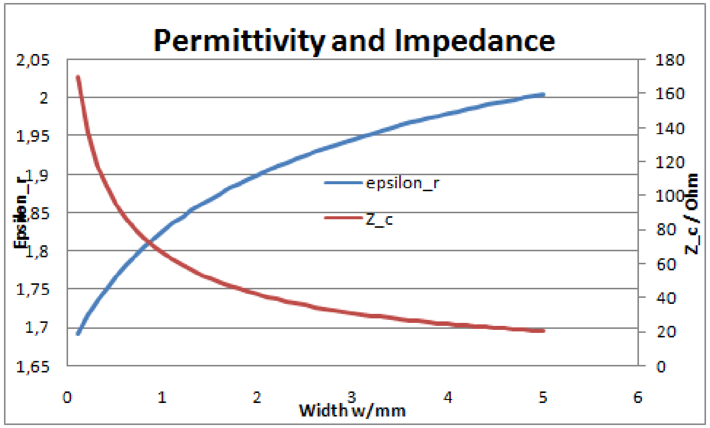
These figures are much used in design of low-pass filters as we will see later.

Micro-Strip-Line

Geometry



Impedance and effective permittivity



($\epsilon_r = 2.2 \quad h = 0.5$ mm)

Micro-Strip Line: Some Remarks

- Relatively easy to handle in PCB
- OPEN is OK, but not perfect (to be seen later)
- SHORT only manufacture-able with via (not ideal, either)
- BENDS/ CURVES should be mitered
- (Quasi-static) Effective design formulas exist, also for buried or multi-layered micro-strip lines
- Watch out for
  - Disturbances in Ground plane
  - Coupling to other lines
  - Disturbances (e.g. shielding cages) in vicinity of the line!
  - substrate may be an-isotropic, i.e. ϵ_r differs with direction.

Calculation of Effective Properties of MS

Effective dielectric constant:

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\sqrt{1 + \frac{12H}{W}}^{-1} + F(\epsilon_r, H) - 0.217(\epsilon_r - 1)\frac{T}{\sqrt{WH}}$$

with

$$F(\epsilon_r, H) = \begin{cases} 0.02(\epsilon_r - 1)(1 - W/H)^2, & \text{for } W/H < 1 \\ 0 & \text{for} W/H > 1 \end{cases}$$

Die characteristic impedance is

$$Z_c = \sqrt{\frac{\epsilon_0 \mu_0}{\epsilon_{eff}}} \frac{1}{C_a}$$

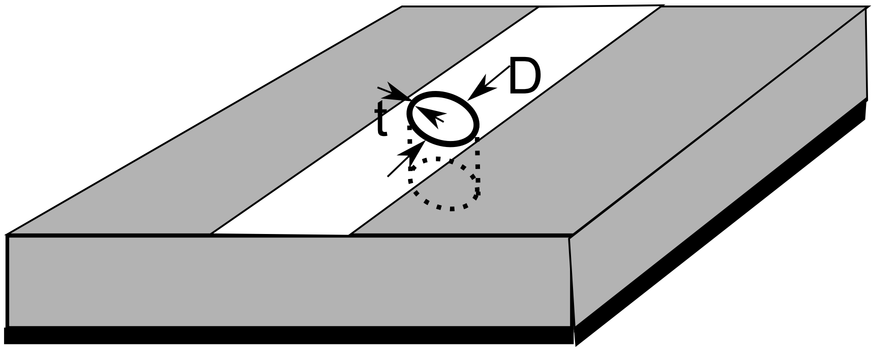
with

$$C_a = \begin{cases} \frac{2\pi\epsilon_0}{\ln(8H/W + 4W/H)} & W/H \leq 1 \\ \epsilon_0 \left[W/H + 1.393 + 0.667 \ln\left(W/H + 1.444\right)\right] & W/H > 1 \end{cases}$$

Micro-Strip: OPEN

- Micro-strip open end is effectively described by length extension or
- Effective capacitance at end of the line
- The wider the strip is, the longer ist the effective extension

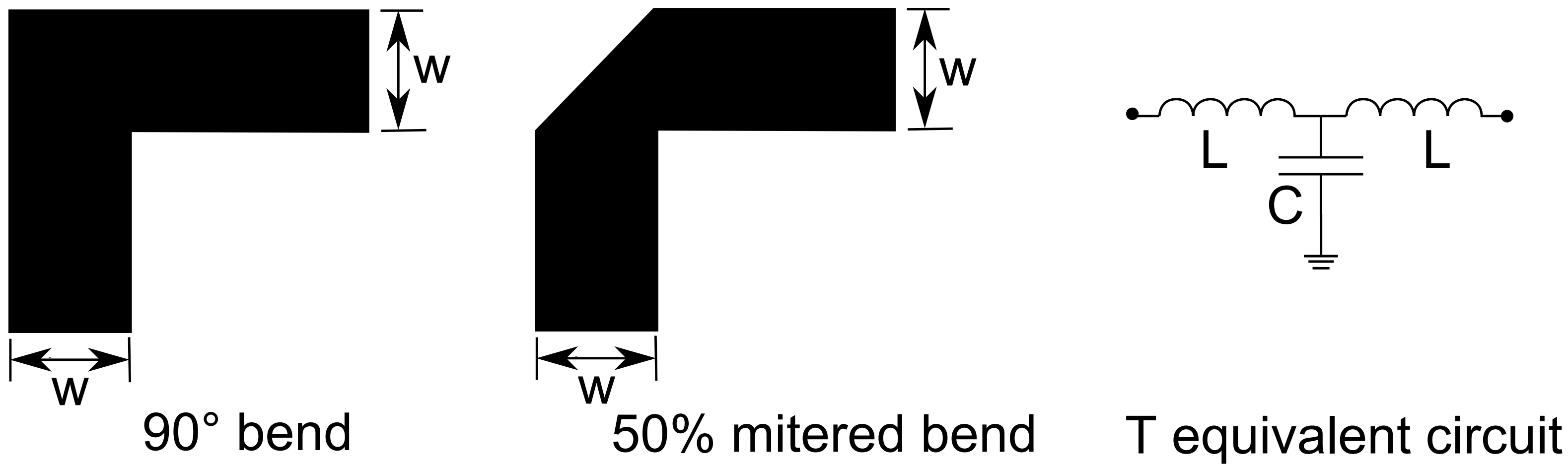
Micro-Strip: SHORT/ VIA



MS via of diameter D and thickness t

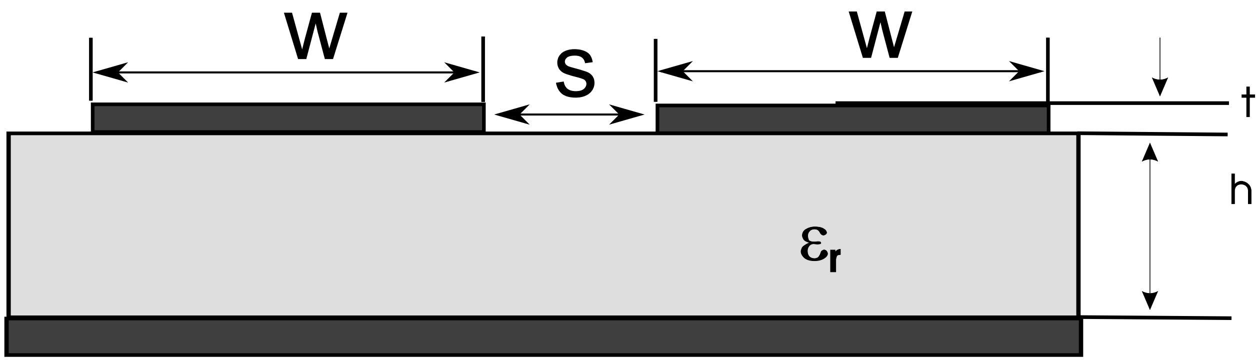
- Micro-strip via is described by series resistance and inductor to ground
- Series resistance: Depends on DC res. of structure (i.e. specific resistance of material) and on skin-effect (current only at outer boundary of via
- Inductance is dominant

Micro-Strip: BENDS



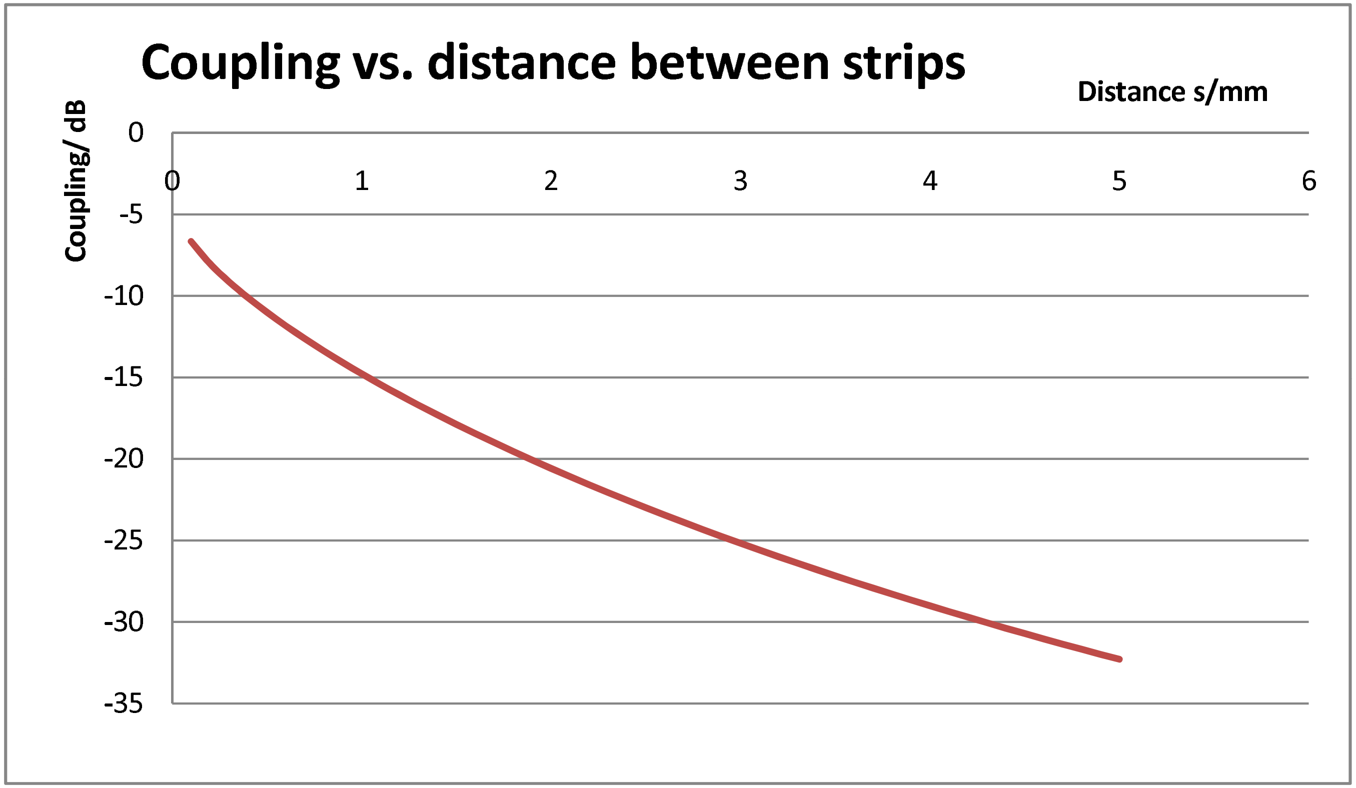
- Micro-strip bend should be conducted as mitered bends
- Miter acts as “Mirror” so that the wave “sees” the line going on straight
- Equivalent circuit is a T-diagram with L and C

Micro-Strip: COUPLED LINES



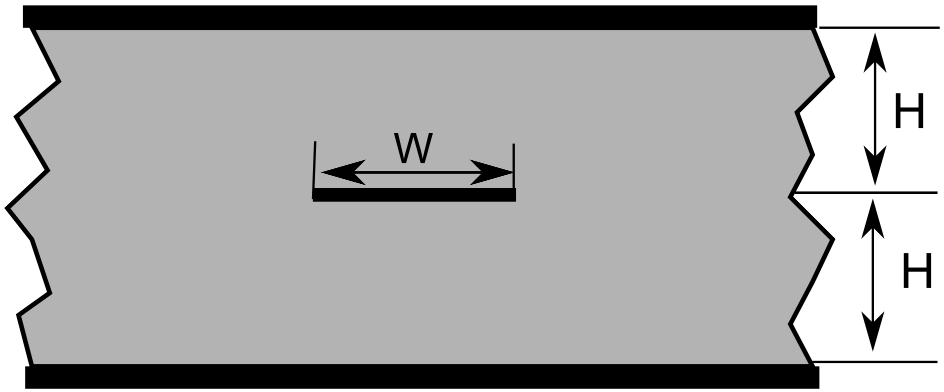
- Supports two modes: Even and Odd thus supports two different impedances: Z_o, Z_e.
- Coupling between lines $C = \frac{Z_e - Z_o}{Z_e + Z_o}$
- Characteristic impedance of line-system is geometric mean of even and odd mode $Z_0 = \sqrt{Z_e Z_o}$
- Usage in coupler structures!
- Unwanted parasitics: Two micro-strip lines in parallel are couplers, thus: Do not get too close

Micro-Strip: Coupled Lines



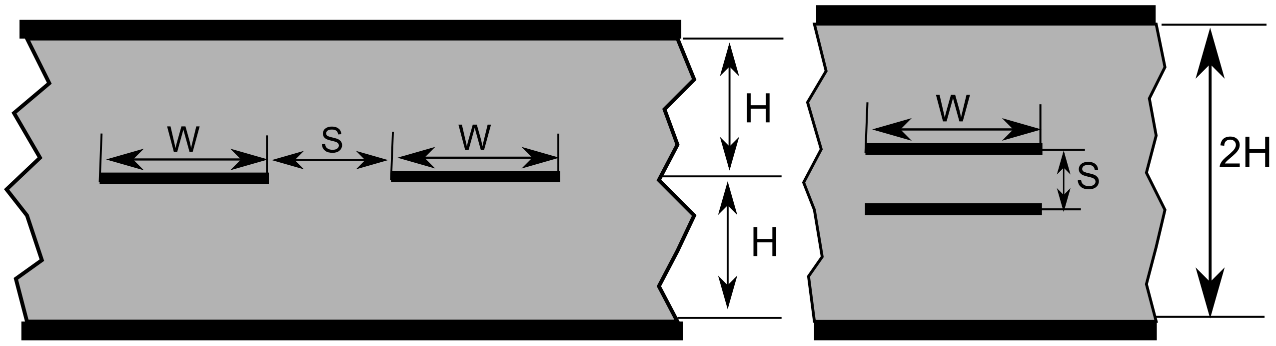
Micro-strip couples lines ($w = 1$mm, $h = 1$mm,ϵ_r=9.7)
Coupling for even $2w$ spacing between the lines is still significant!

Stripline



- Shielded to both: top and bottom
- Can be almost fully shielded when vias are used at sides
- Electrical field better confined due to higher effective permittivity ($\epsilon_{eff} = \epsilon_r$), low dispersion
- Requires at least three layers

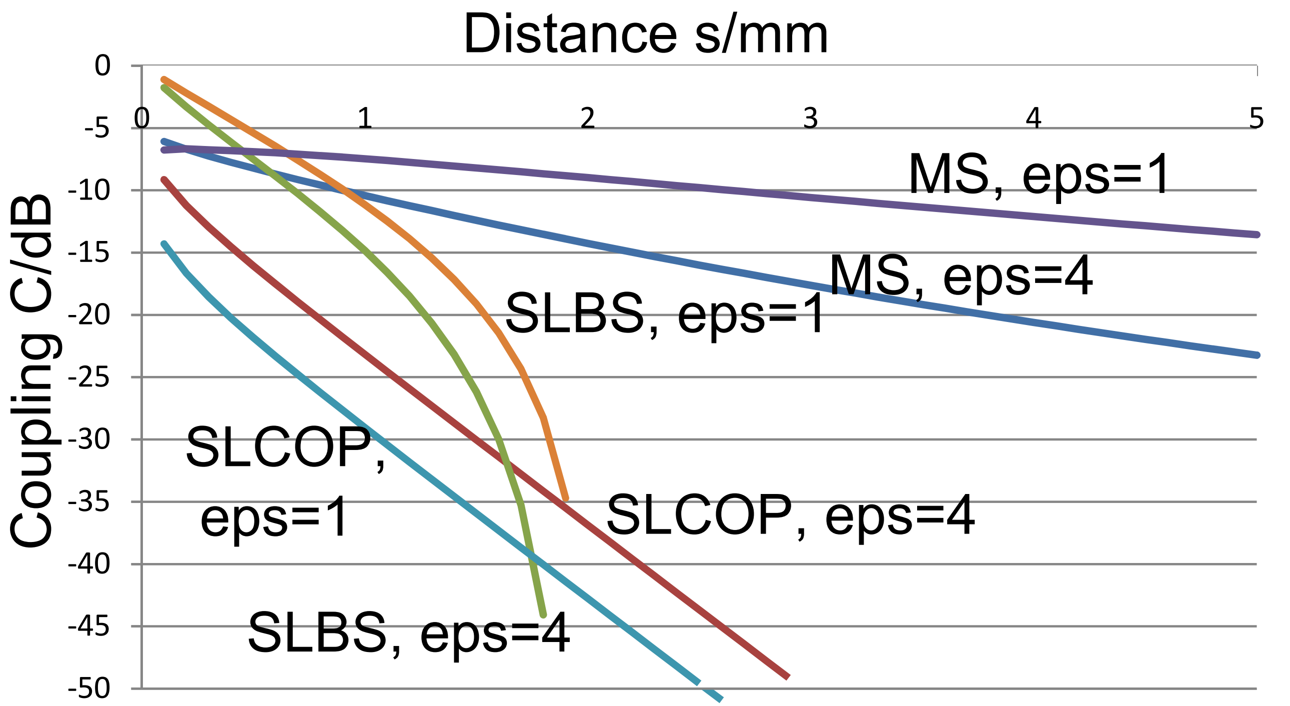
Stripline: Coupled



Coplanar (a) and broadside (b) coupled striplines

- Coplanar: All in one layer
- Broadside: Lines in different layers

Coupling of Practical Lines



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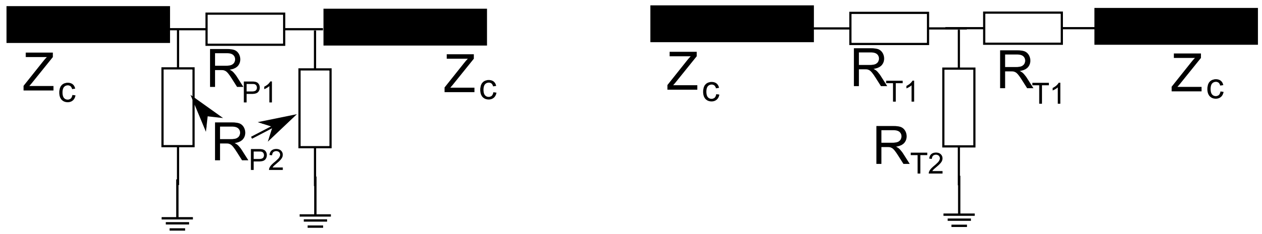
- Micro-strip line has highest coupling, Coplanar coupled stripline best de-coupling

Parameters: $h = 1\,$mm, $Z_0 = 50\,\Omega$ for (centered) strip, $\Rightarrow$

Name	Short	$(w/mm, \epsilon_r = 1)$	$(w/mm, \epsilon_r = 1)$
Strip, broadside	SLBS	2.9	1.0
Strip, coplanar	SLCOP	2.9	1.0
Micro-strip	MS	5.0	2.1

Passive Components and Filters

Simple Resistive Attenuators



For an attenuation ratio $c = u_2/u_1$ the resistors are to be dimensioned as (equal characteristic impedances Z_c at both sides)

- T-circuit $R_{T1} = \frac{1-c}{1+c} Z_c,\ R_{T2} = \frac{2c}{1-c^2} Z_c$
- Π-circuit calculated from T-model via star-triangular transform: $R_{P1} = R_{T_1} \frac{2R_{T1}+R_{T2}}{R_{T2}},\ R_{P2} = R_{T1} + 2R_{T2}$
- Note: There is power to be absorbed in the attenuator, thus thermal issues may occur!
- See next slide for example values

Attenuator: Resistive Values

Resistor values for attenuators at $Z_c = 50\Omega$

Att./ dB	U_1/U_2 (lin)	R_{P1}/Ω	R_{P2}/Ω	R_{T1}/Ω	R_{T2}/Ω
0.25	0.97	1.4	3474.6	0.7	1736.9
0.5	0.94	2.9	1737.7	1.4	868.1
1	0.89	5.8	869.5	2.9	433.3
2	0.79	11.6	436.2	5.7	215.2
3	0.71	17.6	292.4	8.5	141.9
5	0.56	30.4	178.5	14.0	82.2
10	0.32	71.2	96.2	26.0	35.1
20	0.10	247.5	61.1	40.9	10.1
30	0.03	789.8	53.3	46.9	3.2

Filter Design

Filter needed for

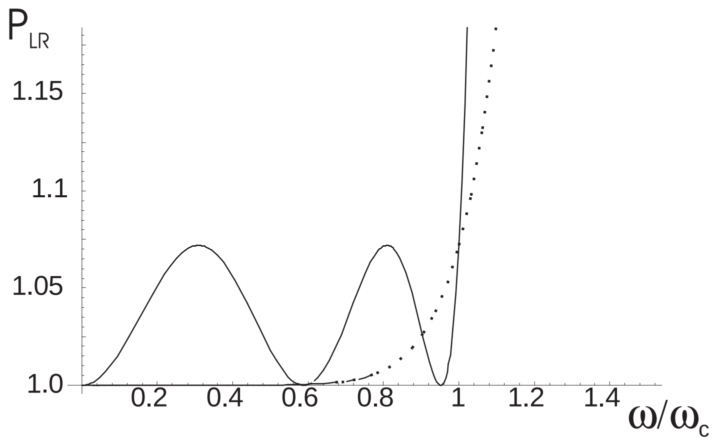
- RF-RX: RF selection (reject blockers, limit bandwidth, thus minimize noise bandwidth)
- RF-TX: Keep spectrum clean, do not disturb others! Shape spectrum.
- IF: Separate wanted and unwanted signal (e.g. LO rejection), reject image frequencies

Filter development

- Systematic approach to design frequency response
- Translate from prototype (low-pass) filters wanted shape (e.g. bandpass)
- Translate into physical structures. Here: transmission-lines

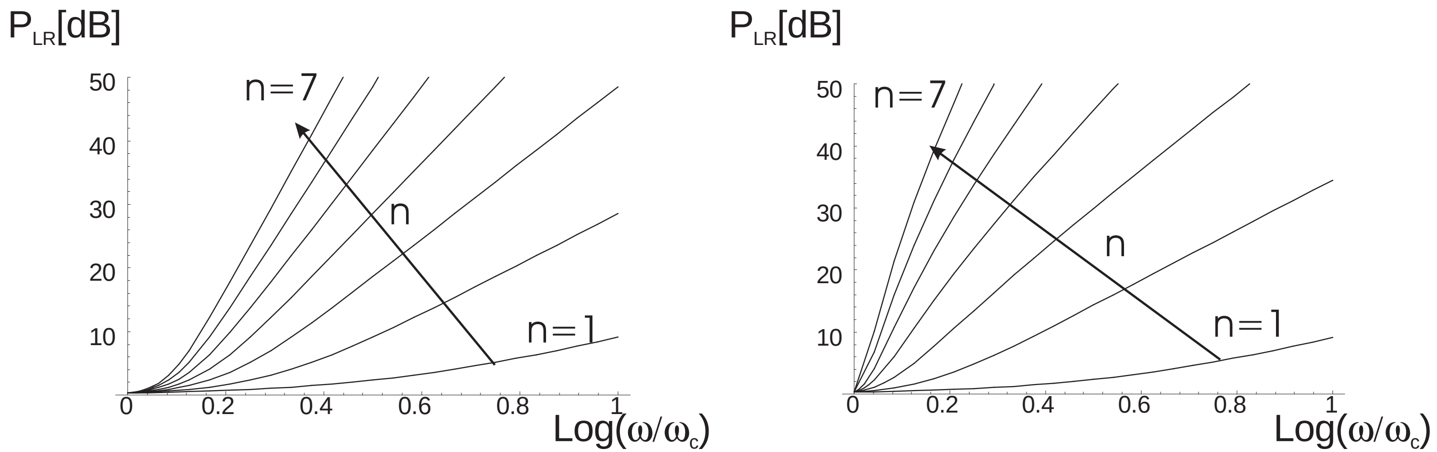
Prototype-Filters

- Starting point: Power-Loss-Ratio $P_{LR} = \frac{1}{1-\rho^2} = \frac{1}{1-\Gamma\Gamma^*}$
- Express with well defined ratio of polynomials:
  $P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$
- Butterworth-filter for $N(\omega^2) = 1$ and $M(\omega^2) = k^2(\omega/\omega_c)^{2N}$
- Chebycheff-filter for $N(\omega^2) = 1$ and $M(\omega^2) = k^2 T_N^2(\omega/\omega_c)$ for Chebycheff-characteristic. T_N: Chebycheff-Polynomial order N



Characteristics for Butterworth- (- - -) und Chebycheff-low-pass (—) order 5

Prototype Filter Characteristics

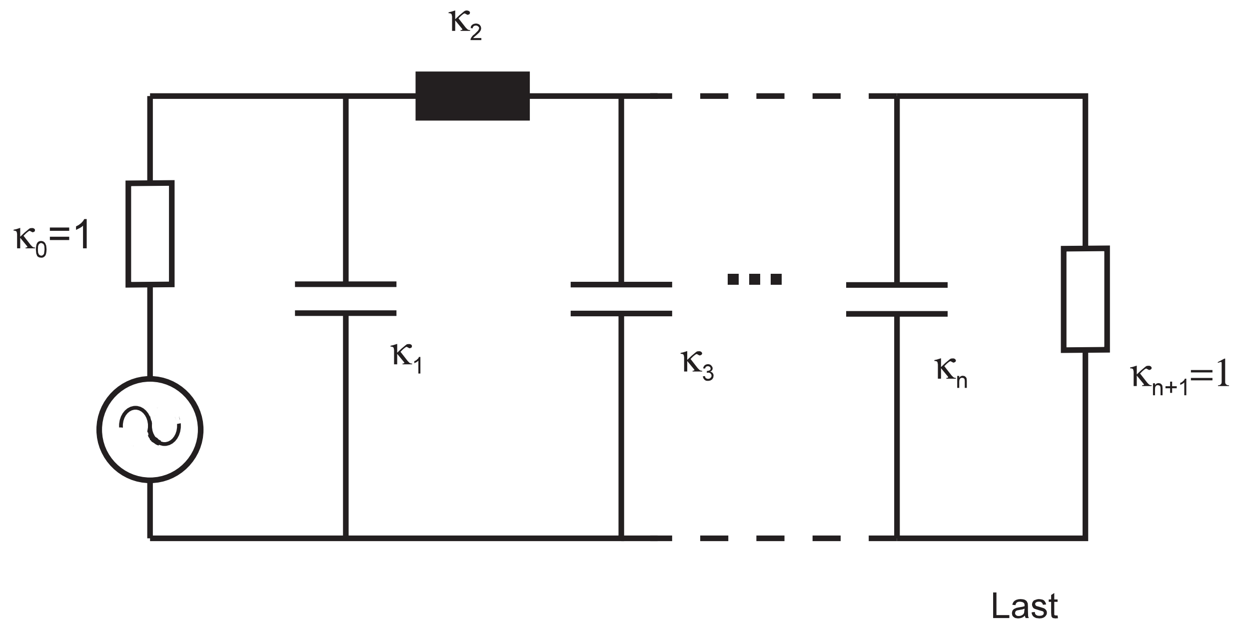


(a) (b)

Rejection-characteristics for different order filters ripple $\rho_{max} = 1$ dB. (a) Butterworth, (b) Chebycheff.

- Chebycheff:
  - Highest possible (polynomial) selection
  - Ripples in pass-band
- Butterworth:
  - Maximum flat in passband
  - Fairly low rejection

Filter Prototype Structure



Low-pass prototype, starting with cap.

Element Values: Butterworth

n	1	2	3	4	5	6	7
κ_1	2	1.4142	1	0.7654	0.6180	0.5176	0.4450
κ_2	1	1.4142	2	1.8478	1.6180	1.4142	1.2470
κ_3		1	1	1.8478	2	1.9319	1.8019
κ_4			1	0.7654	1.6180	1.9319	2
κ_5				1	0.6180	1.4142	1.8019
κ_6					1	0.5176	1.2470
κ_7						1	0.4450
κ_8							1

Prototype Elements for Butterworth filter

- All orders terminate in 1 $\Rightarrow$ All orders lead to matched filter
- Elements calculated with $L_k = \frac{\kappa_k Z_0}{\omega}$ and $C_k = \frac{\kappa_k}{\omega Z_0}$
- ω is 3 dB corner frequency, Z_0 impedance level (here: 50 Ω)

Element Values: Chebycheff

n	1	2	3	4	5	6	7
κ_1	1.018	1.822	2.024	2.099	2.135	2.155	2.166
κ_2	1	0.685	0.994	1.064	1.091	1.104	1.112
κ_3		2.660	2.024	2.831	3.001	3.063	3.093
κ_4			1	0.789	1.091	1.152	1.174
κ_5				2.660	2.135	2.937	3.093
κ_6					1	0.810	1.112
κ_7						2.660	2.166
κ_8							1

Table of prototype values for Chebycheff low-pass with $\rho_{max} = 1$ dB

- Only odd orders terminate in 1, thus only they can be matched to 50 Ω

Recipe for Low-Pass-Filter

1. Select corner frequencies ω, maximum loss (ripple) ρ_{max} in pass-band, selectivity
2. Selection of maximum ripple determines which table (from elaborate literature) to use. Standard values for ρ_{max} are usually tabulated
3. Selectivity requirements determine order. Evtl. use graphics for this
4. Take prototype (normalized) values κ_n
5. Carry values over to physical network with $L_k = \frac{\kappa_k Z_0}{\omega}$ and $C_k = \frac{\kappa_k}{\omega Z_0}$
6. If transmission line-design wanted, use formulae for short lines and calculate length $l_L = \frac{\lambda_L}{2\pi} \arcsin(\omega C Z_L)$ for low impedance $(_L)$ and $l_H = \frac{\lambda_H}{2\pi} \arcsin\left(\frac{\omega L}{Z_H}\right)$ for high impedance $(_H)$ section.

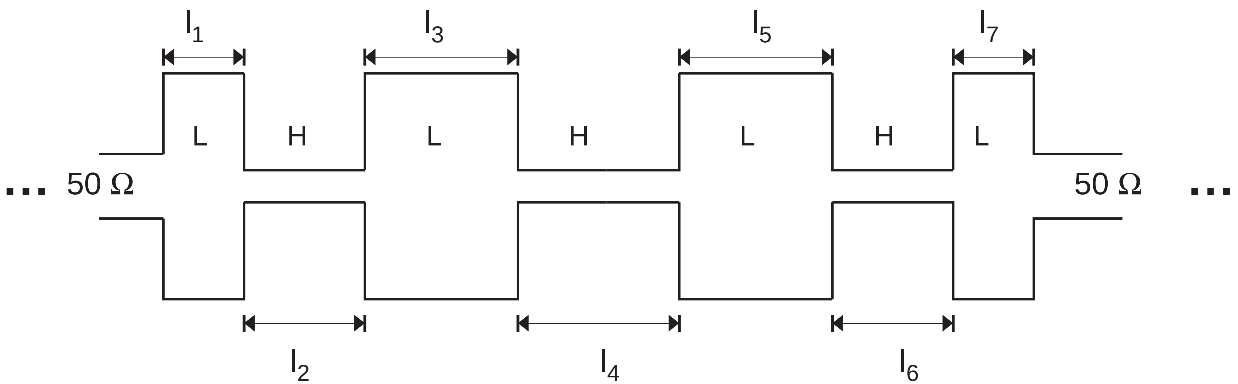
Example: Low-Pass in Micro-strip Technology

Design requirements

- Butterworth, order ≥ 5 (we will chose 7)
- Low-pass up to 1 GHz, high rejection for $f \approx 10$ GHz
- Design for $50\,\Omega$ in microstrip-technology, on RT-Duroid 5880, $\epsilon_r = 2.2$, h=0.508 mm
- Chose impedances (should be manufacturable, only supporting one mode and still as far apart as possible), $Z_H = 120\,\Omega,\ w_H = 0.27$ mm; $Z_L = 15\,\Omega,\ w_L = 7.3$ mm
- Compensation done empirically (just scale length of lines)

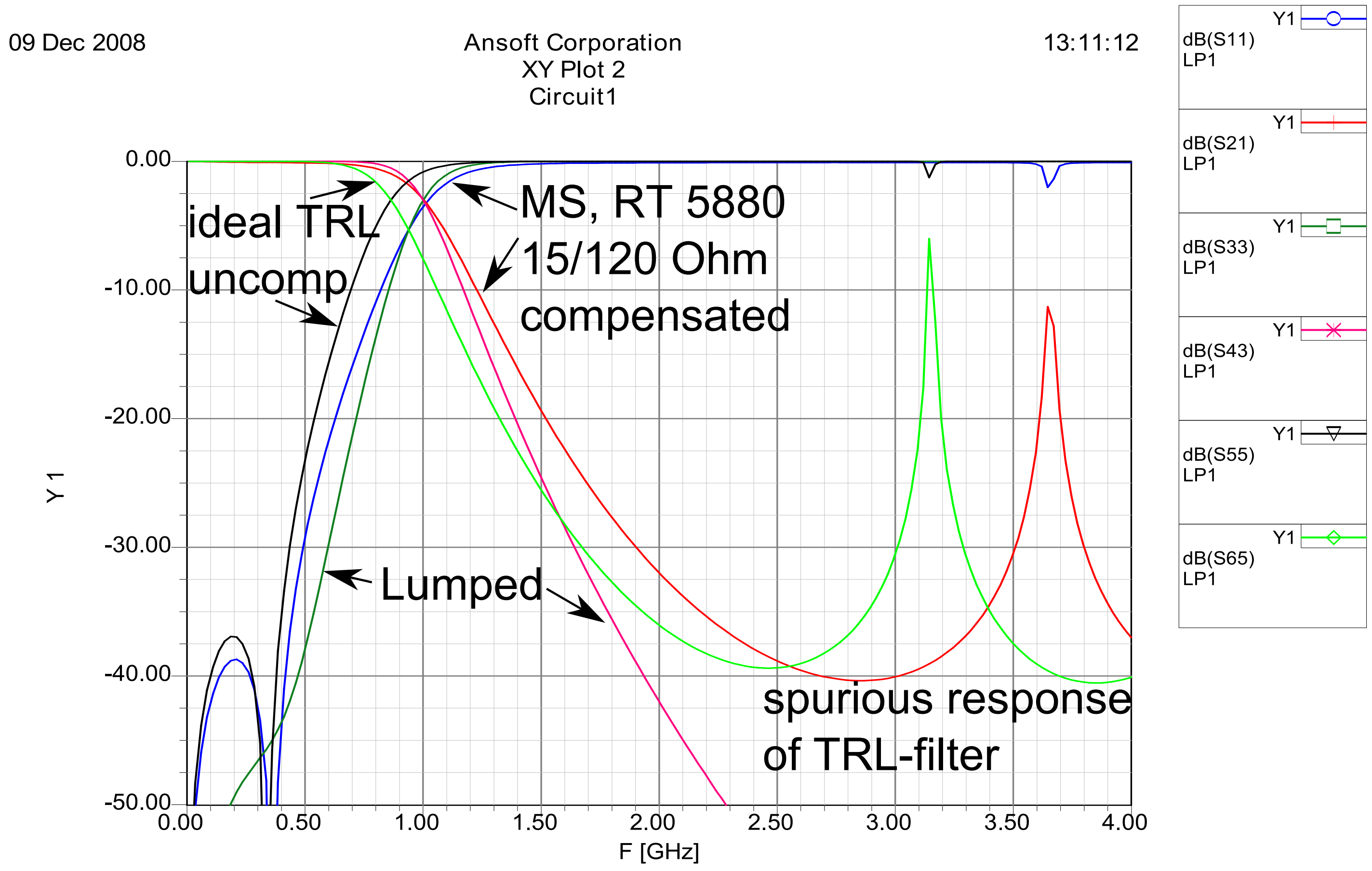
Element	Value	el. l uncomp.	phys.l/mm comp.
C_1	1.4 pF	7.7°	3.6
L_1	9.9 nH	31°	16.4
C_2	5.7 pF	33°	15.6
L_2	16 nH	56°	29.6
C_3	5.7 pF	33°	15.6
L_3	9.9 nH	31°	16.6
C_4	1.4 pF	7.7°	3.6

Low-pass-Filter Layout



Micro-strip-layout of a Butterworth low-pass.

Low-Pass-results



Design of Distributed Band-Pass-Filter

- General approach is to transform prototype low-pass to band via a transformation function (for band-pass:
$b(\omega) = \frac{\omega_0}{\Delta\omega}\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)$
- Bandpass with coupled line section design equations:

$$
\begin{aligned}
\omega_0 &= \sqrt{\omega_1 \omega_2} \\
\Theta_1 &= \frac{\pi \omega_1}{2\omega_0} \quad , \\
P \sin \Theta_1 &= \frac{K'_{10}}{\sqrt{\frac{1}{2}\tan \Theta_1 + {K'_{10}}^2}} \quad , \\
s &= \frac{1}{\frac{1}{2}\tan \Theta_1 + {K'_{10}}^2} \quad .
\end{aligned}
$$

Design of Distributed Band-Pass-Filter

- Values for impedance normalized (to Z_c) impedance inverters:

$$K'_{10} = K'_{n+1,n} = \frac{1}{\sqrt{\kappa_0 \kappa_1}} = \frac{1}{\sqrt{\kappa_n \kappa_{n+1}}} \quad ,$$

$$K'_{k,k+1} = \frac{1}{\sqrt{\kappa_k \kappa_{k+1}}} \quad .$$

- Some values come to aid:

$$\hat{N}_{k+1,k} = \sqrt{K'^2_{k+1,k} + \frac{1}{4}\tan^2\Theta_1}$$

- And finally the even- and odd-mode impedances

$$Z_e^1 = Z_e^{n+1} = Z_c(1 + P\sin\Theta_1) \quad ,$$

$$Z_o^1 = Z_o^{n+1} = Z_c(1 - P\sin\Theta_1) \quad ,$$

$$Z_e^{k+1} = Z_e^{n-k+1} = Z_c s(\hat{N}_{k+1,k} + K'_{k+1,k}) \quad ,$$

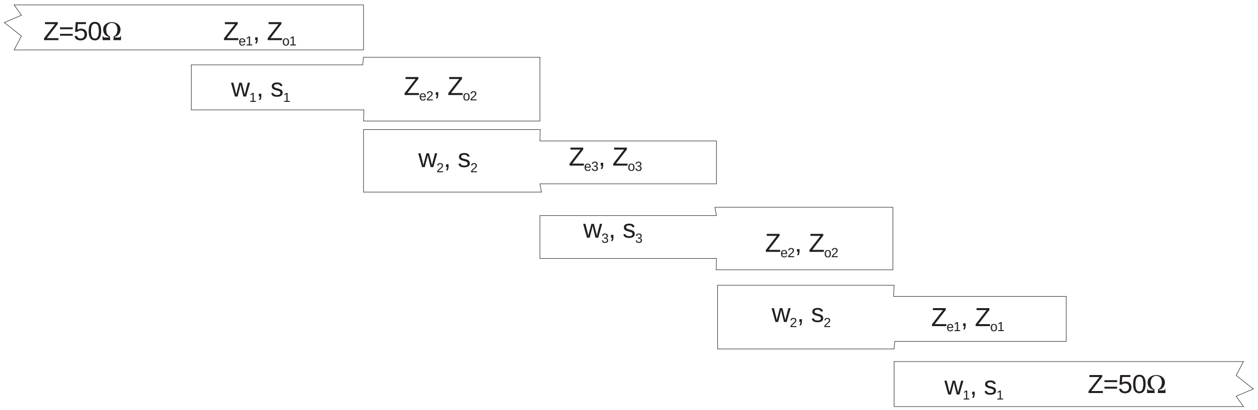
$$Z_o^{k+1} = Z_o^{n-k+1} = Z_c s(\hat{N}_{k+1,k} - K'_{k+1,k}) \quad .$$

Design Results for Bandpass

Chebycheff, 1 dB equal-ripple filter, pass-band 11 ... 12 GHz, on 50 Ω-System, Order $N = 7$ Impedances in Ω, Symmetry

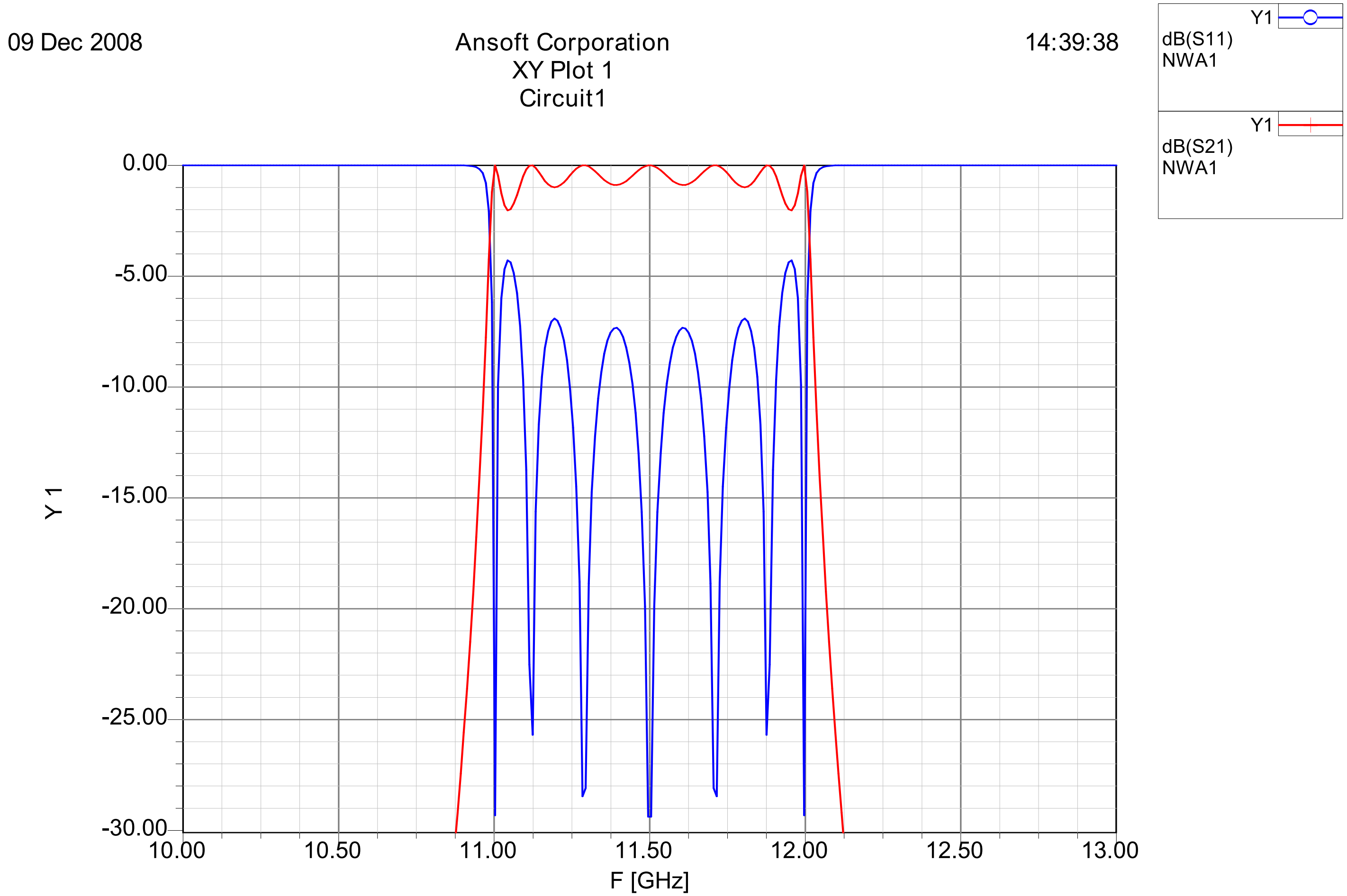
$$Z_{N-k} = Z_k$$

Z_{e1}	62.0	Z_{o1}	37.9
Z_{e2}	51.3	Z_{o2}	43.2
Z_{e3}	50.6	Z_{o3}	43.8
Z_{e4}	50.5	Z_{o4}	43.9



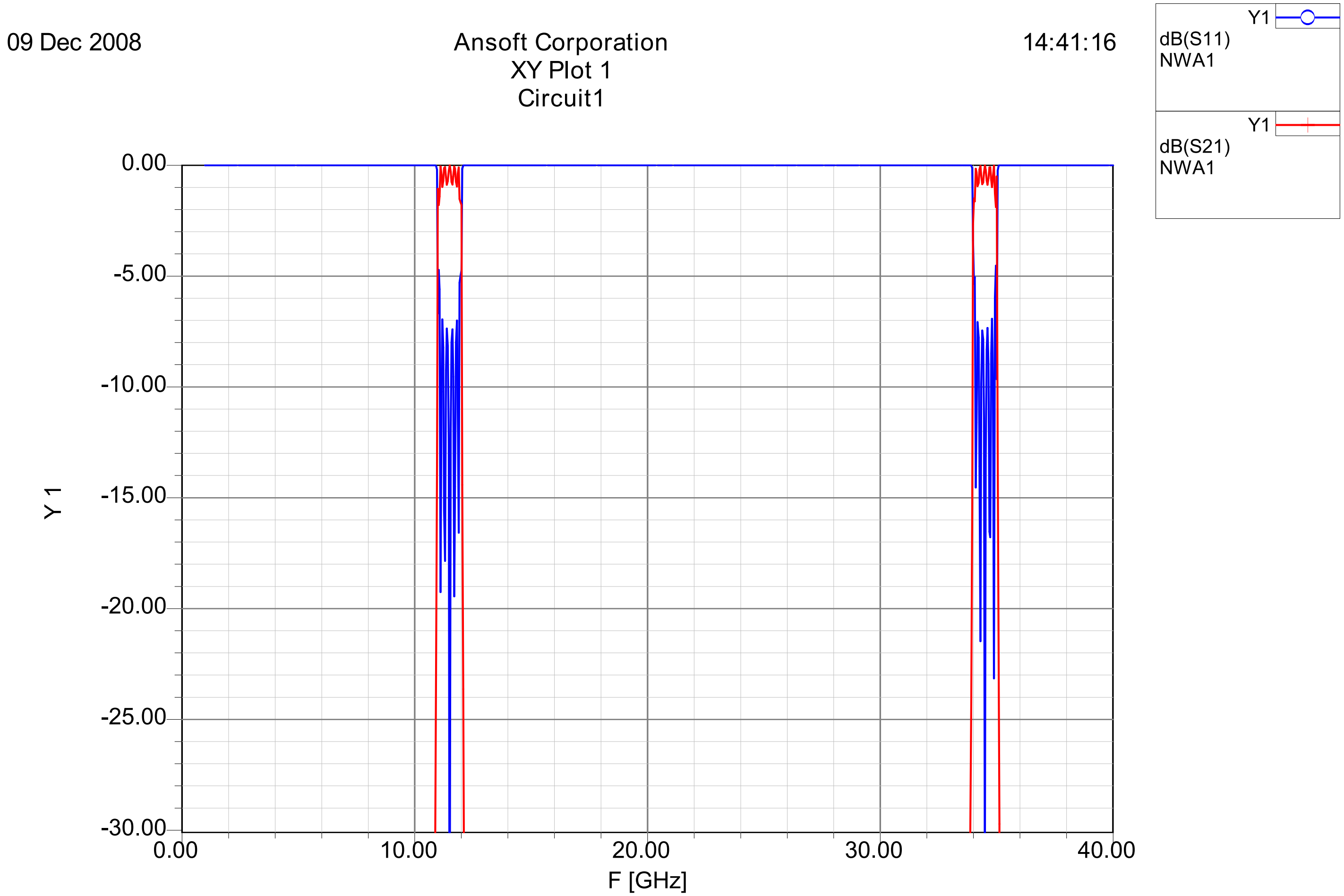
Layout (schematically) of a bandpass filter in microstrip technology

Bandpass Results I



Simulation for bandpass filter (N=7) with ideal coupled transmission-lines (no end-effect, no real lines considered). Observe the number of ripples!

Bandpass Results II



Simulation as above, observe the spurious pass-band at $3f_0$, where the lines have an electrical length of $3\lambda/2$. No passband at length λ!